Centrality Measures for Trade and Investment Networks

Muhammad Mohsin Hakeem and Ken-ichi Suzuki

Graduate School of Economics and Management, Tohoku University, Sendai, Japan

Abstract

A centrality measure for network defines the significance of a single node. Different centrality measures are used to identify important nodes in different types of networks. There are local centralities measures such as degree centrality, and relative centrality measures such as closeness or betweenness centrality. We used diverse centrality measures to analyze the European Union's Trade and Eurozone Investment Network. To identify the changes due to global financial crisis we used trade and investment networks of 2007 and 2011, the year's corresponding to before and after the crisis.

Keywords: Network; Financial Network; Trade Network; Centrality; Influential Nodes; Financial Crisis

1. Introduction

A Network, in its simplest form is a combination of nodes with edges as connections. Networks can be simple or complex; can have few nodes with limited connectivity or large number of nodes with complex connections. Understanding networks and its basic structure attracted attention recently. Current discussions in literature are focused on network structure, functions and their relations, rather real world implications.

A centrality measure for network defines the significance of a single node. Central nodes can leads to faster and wider spreading in complex networks as explained by Borgatti (2005). According to Joseph & Chen (2014b) different centrality measures are used to identify main nodes in different types of networks. He divided the measures into two categories, Radial and Medial Centralities. Limitations of both measures restraint us from generalization, rather particular measure must be used for certain type of network. Recent issues of centrality measures and relevance with financial networks is discussed by Joseph & Chen (2014a), Lin-yuan (2010) and Tonzer (2015) in their respective work.

2. Centrality Measures

The concept of centrality is not new for networks. Centrality measures explain the importance of single or cluster of nodes. There are indeed many definitions of importance of node correspondingly many centrality measures for different types of network. Centrality measures mainly include *degree centrality, eigenvector centrality, hubs and authorities, closeness centrality* and *betweenness centrality*. Financial networks are unique with respect to their clearing mechanism, flow dynamics and other characteristics. We apply range of centrality measures on real world financial (investment) and trade networks, to identify relevant ones, for these types of networks. Network of portfolio investment within European Unions and its trade graphs across different member and non-member countries are used to identify predominant nodes. Centrality measures mentioned above are explained as follows.

3. Degree Centrality

The simplest and earliest centrality measure in a network is the degree of a node, the number of edges connected to it. In directed networksf nodes have both an in-degree and an outdegree, and both may be effective if used in the appropriate circumstances. Although degree centrality is a simple centrality measure, it can be very insightful. In a financial network, for instance, the financial institution or a node connected to all other nodes can have much more influence on other nodes as well as the resilience of whole network. Degree centrality is calculated by using the Freeman's (1979) general formula for centralization. This is as follows:

$$C_D = \frac{\sum_{i=1}^{g}\left[C_D(n^*)-C_D(i)\right]}{[(N-1)(N-2)]}$$ .... (1)

Where *degree centrality* "C_D" is calculated by using the maximum value, while N represents the number of nodes within that particular network.

4. Eigenvector Centrality

Eigenvector centrality is a likely extension of degree centrality. As we understood that degree centrality is the about getting a centrality point for every network neighbour a node have. But every node is distinctive having particular characteristics associated with it only. So a nodes importance in a network would increase if it has connections with other *themselves important* nodes. This is the concept behind eigenvector centrality. Instead of awarding nodes just one point

for each neighbour, eigenvector centrality gives each node a score proportional to the sum of the scores of its neighbours. Bonacich (1987) explained the *eigenvector centrality* as follows,

$$C(\alpha,\beta) = \alpha(I - \beta R)^{-1}R1(2) \quad \text{... (2)}$$

Where α is a scaling vector, that is used to normalize score, whereas β reflects the extent to which centrality of a node can be weighted with respect to particular characteristics or under consideration conditions of networks such as size of node, or total flow. **R** is the adjacency, **I** is the identity, and **1** is an all ones matrix. By changing the values of β one can adjust the characteristics required to affect the eigen value centrality. β equals to zero simply gives degree centrality.

5. Hubs and Authorities

For directed networks, we can give centrality measures another dimension by using *Hubs* and *Authorities*. Up till now the nodes can have higher centrality if they have incoming edges with high centrality. However, in some networks it is fine to give a node high centrality if it *points to* others with high centrality. So if a node is connected to another authoritative or important node it can have higher level of centrality due to this connection. For example, subsidiary of an investment firm, bank or hedge fund can have higher centrality due to connection with the parent firm. The nodes which direct to authoritative nodes are known as hubs though an authority can also be a hub or vice versa. This measure is exclusive for directed networks, as other networks do not have pointing edges.

The idea of *authority* and *hub* centrality was introduced by Kleinberg (1999) and then later developed into a centrality algorithm called *hyperlink-induced topic search* or *HITS*.

For ranking purpose,

$$\forall p, auth(p) = 1 \text{ and } hub(p) = 1 \quad \text{... (3)}$$

There are two types of updates, Authority Update Rule and Hub Update Rule, to calculate the scores for each node repeated iterations of both rules are applied. A *k-step* application of the Hub-Authority algorithm would require the *k* times application of the Authority Update rule and then Hub Update Rule.

Both rules are as follows:

$\forall p$ we update *auth(p),*

$$auth(p) = \sum_{i=1}^{n} hub(i) \quad\text{.. (4)}$$

$\forall p$ we update $hub(p)$,

$$hub(p) = \sum_{i=1}^{n} auth(i) \quad\text{.. (5)}$$

Where n is the total number of nodes connected to p.

6. Closeness Centrality

This centrality measure is totally different, as it measures the mean distance from one node to other nodes. It is the concept of geodesic path, - the shortest path between two nodes-. Closeness centrality has small values for nodes that are separated from others by only a short geodesic distance on average. Such nodes might have better access to information at other nodes or more direct influence on other nodes. In a financial network, for example, a financial institution with lower mean distance to others might have better access to liquidity and important financial information. Closeness centrality is a very natural measure of centrality and is often used in different types of network studies. Closeness is based on the length of the average shortest path between a vertex and all vertices in the graph

$$C_c(i) = \left[\sum_{j=1}^{N} d(i, j) \right]^{-1} \quad\text{.. (6)}$$

Whereas Normalized Closeness Centrality is,

$$C_{c}{}^{\flat}(i) = (C_c(i))/(N-1) \quad\text{.. (7)}$$

Where, d is the distance between node i and j, while N refers to the number of nodes within network. The concept of closeness centrality is not new and is based on the foundations laid by Bavelas (1950), who used this as multiplicative inverse of farness.

7. Betweenness Centrality

Betweenness Centrality is another different centrality concept; it measures the extent to which a node lies on path between other nodes. It quantifies the number of times a node act as a bridge along the geodesic path between two other nodes. To understand the concept we need to

look at the financial or trade network. In financial network there is a flow of money or liquidity and in trade network there is a flow of goods between two or more nodes. If the flow of goods or money needs to pass through specific node to reach their destination then that particular node has enormous power to influence the counter parties. There can be several ways to reach destination but geodesic paths are designed to be efficient and cost effective. So nodes lying on that path have higher betweenness centrality and influence on the whole network. A bank or investment firm with higher betweenness centrality must be stable and strong for the network's resilience.

The idea of betweenness is presented by Freeman (1977), although he mentioned some unpublished works by other authors on this particular issue. Mathematically, we can express the betweenness for a general network by $g_{jk}(i)$ to be the number of geodesic paths from j to k that pass through i. And we define g_{jk} to be the total number of geodesic paths from j to k. Then the betweenness centrality of node i is

$$C_B(i) = \sum_{j<k} g_{jk}(i)/g_{jk} \quad\text{.. (8)}$$

Usually normalized by

$$C_B^{\,\prime}(i) = C_B(i)/[(n-1)(n-2)/2] \quad\text{.. (9)}$$

Where n is the total number of nodes within network.

Though there are other centrality measures used in different networks, but above mentioned measures are the most relent with financial and trade networks, and would be used in analysis.

8. Why Centrality Matters

We introduced different centrality measures and their calculation techniques. But do we need all these measures to see through the networks. The answer is not simple, rather complex but we do need all of these and other measures to understand the networks in better and convenient way. Centrality measures can be divided into local measures such as degree centrality and relative measures such as closeness and betweenness centrality. These measures are designed to explain the nodes characteristics in different ways. In finance, investment and trade we need to focus on every important node. These measures enlighten us about the importance of every node by

showing relevant statistics. As in large networks attention can't be given to every single node, one can easily find the important and relevant nodes with the help of these measures.

Every measure has certain properties and characteristics that can explain about the underlying number within networks. These properties are explained in earlier literature; new readers may wish to explore some basic concepts before moving to discussion.

9. Real World Networks

We selected two different networks for centrality analysis, which are as follows,

1. European Union Trade Network
2. Eurozone Investment Network

European Union Trade Network (EUTN) includes 28 European Union (EU) members and its 3 biggest trading partners which are China, US and Japan. So trade network contain 31 countries. The data is obtained for 2 different years, 2007 and 2011, to observe the changes within network due to global financial crisis. The time period is vital to observe changes in world trade and economy. Trade networks are obtained from Direction of Trade Statistics (2015) also known as DOTS.

European Union Investment Network (EIN) consist 17[1] Eurozone member states and international organizations. These international organizations are represented by single node with cumulative investment figures for each member state. The data is obtained for 2007 and 2011. As explained earlier that selection of network years is due to global financial crisis and its huge impact on global trade and investment networks. Investment networks are obtained from Coordinated Portfolio Investment Survey (2015) also known as CPIS. Both databases are compiled by International Monetary Fund (IMF) data library.

10. EU Trade Network (EUTN)

Trade network of European Union and its 3 big trading partners, China, US and Japan is shown in Figure 1 (2007) and 4 (2011). Network visualization revels that trade network remains the same in terms of its structure. Though its weights are different and volumes are increased with the passage of time. Another important and visible issue is about linkages among nodes. One can notice that big trading partners have huge influence on the whole network due to attained centrality. Big 3 trading partners of EU are strongly interlinked with themselves and every member

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[1] Eurozone had 17 member states during our analysis period which is from 2007-2011.

of EU, and can transmit shocks to every single node in the network. *Figure 1: EU Trade Network 2007 (@DOTS2015)*
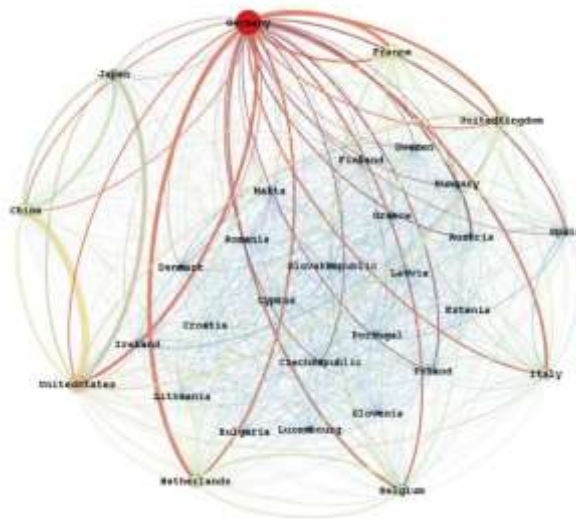

Figure 2 and 3 shows the centrality distribution of 2007 trade network. If we observe the centrality distribution for 2007 network we would realize that this network is strongly connected having highest possible density numbers for a graph such as 0.99. Centrality distribution show the same phenomena as node are acting in uniform and cumulative way. Difference of edge weights is there which one can notice from degree distributions. These distributions also revels the scales of network flows from nodes as node size varies a lot, so is the flow from it. These strong connection and interlinks can be vital for resilience but fatal for transmission of shocks. In other words the network as a whole can be more resilient to the outer shocks but can transmit it to every last node, and may not be able to contain it till specific cluster.

Size of nodes are as per the degree centrality distribution, so if we look at the counties then Germany appears to be the most connected country with the whole European Union with respect to weighted degree distribution. We can find the cluster of countries or big nodes having higher weights of edges such as France, UK, Italy, Spain and Netherland, while the rest smaller nodes, and lower weights edges. The visible cluster as made for better understanding as whole network is unique and complete so technically it only have one cluster of itself.

Figure 4 corresponds to trade network of 2011, the visible networks filter out the smaller edges to show the better picture of larger nodes and connections. Figure 5 and 6 shows the

centrality distribution metrics of 2011 trade network. If we compare 2007 and 2011 trade network centrality metrics we can find large number of similarities.

Figure 2: Centrality Distribution Metrics of EU Trade Network 2007
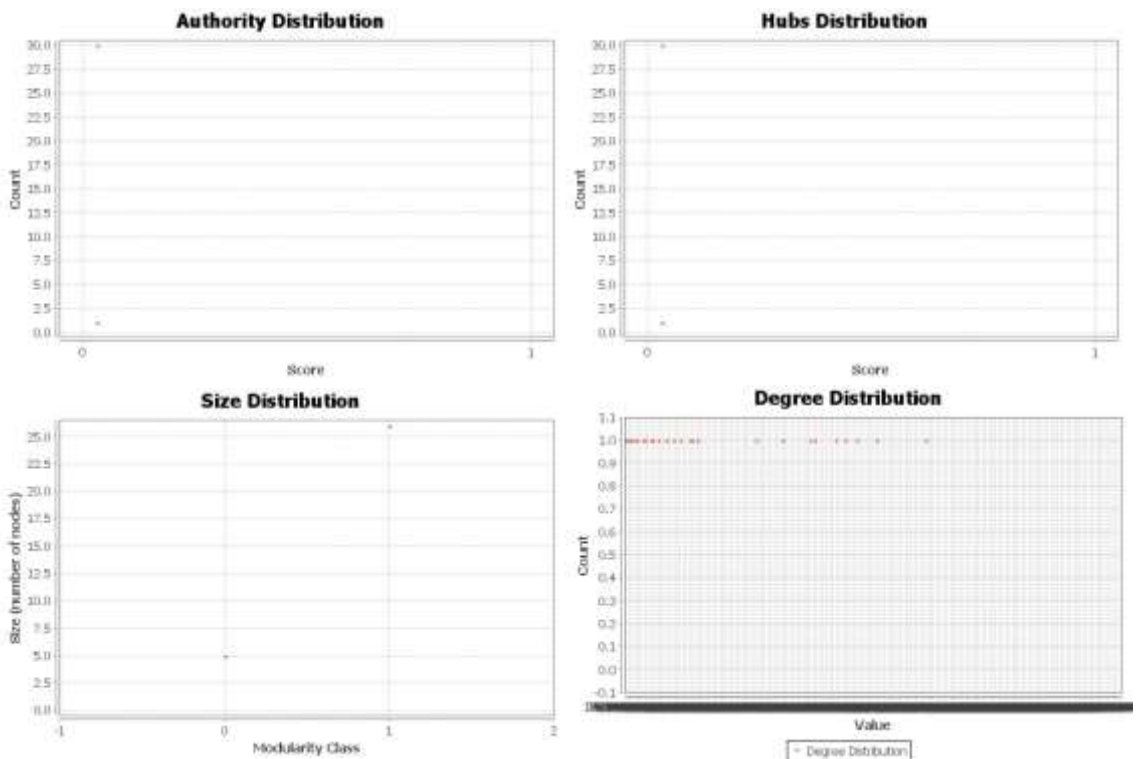
Besides weighted degree distribution, all other distributions are similar, such as betweenness, hubs and authority, closeness, eigenvector and degree centrality. The centrality distribution similarities explain a lot about dynamics of trade network. Complex, stable, resistant with large weights, these connections cannot be broken. It can be due to the complexities involved in trade of goods and services as it's not the liquidity only but huge quantity of supplies that moves between nodes. By comparing both trade networks and centrality measures, we believe that financial crisis in 2008 was not able to change much about trade network in these years as both structures are complete with similar weights.

Figure 3: Centrality Distribution Metrics of EU Trade Network 2007
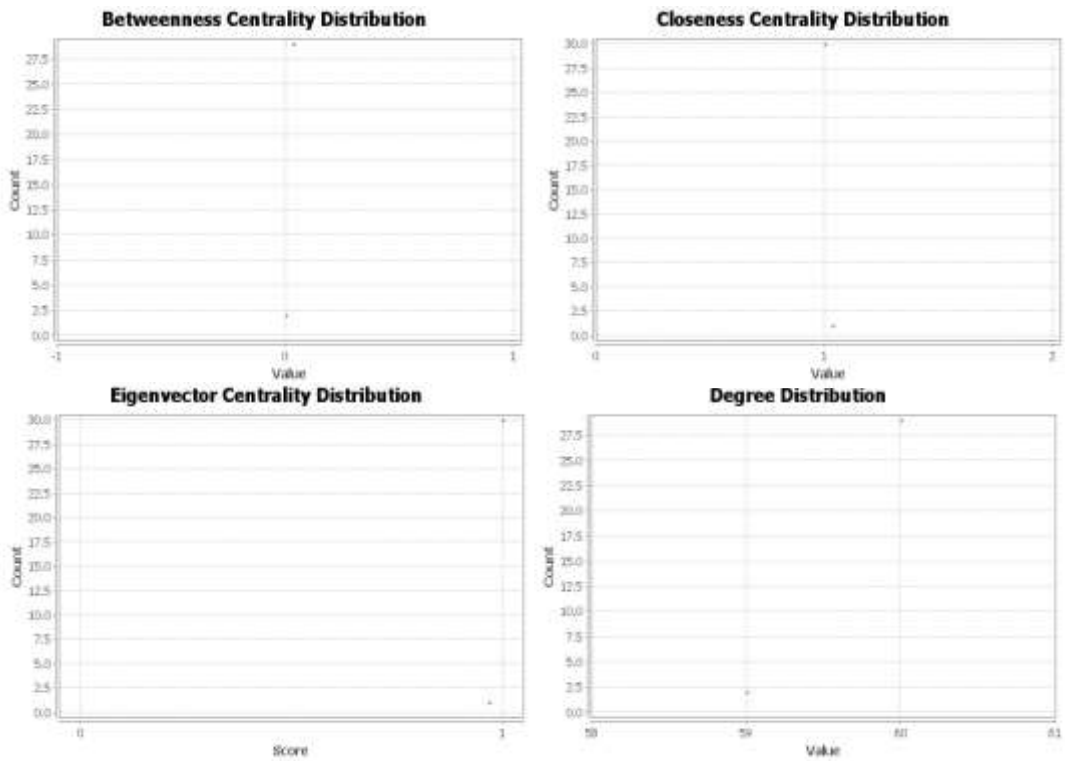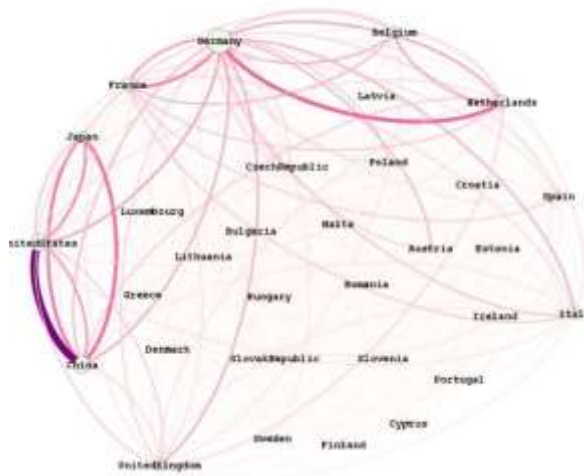
Figure 4: EU Trade Network 2011(@DOTS2015)

Figure 5: Centrality Distribution Metrics of EU Trade Network 2011
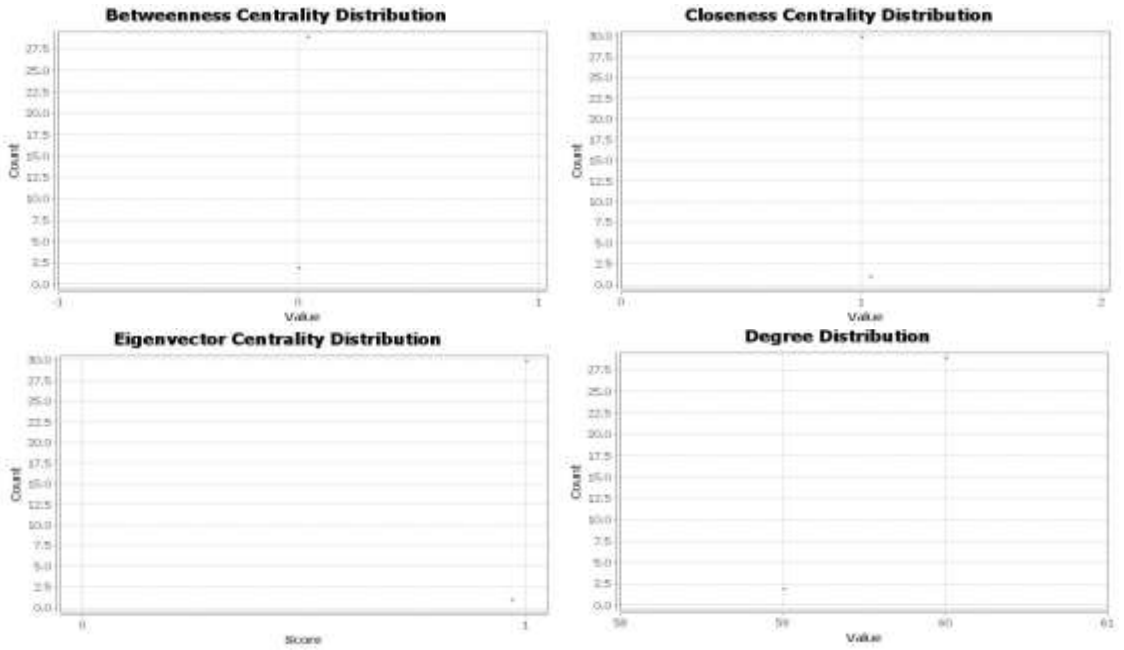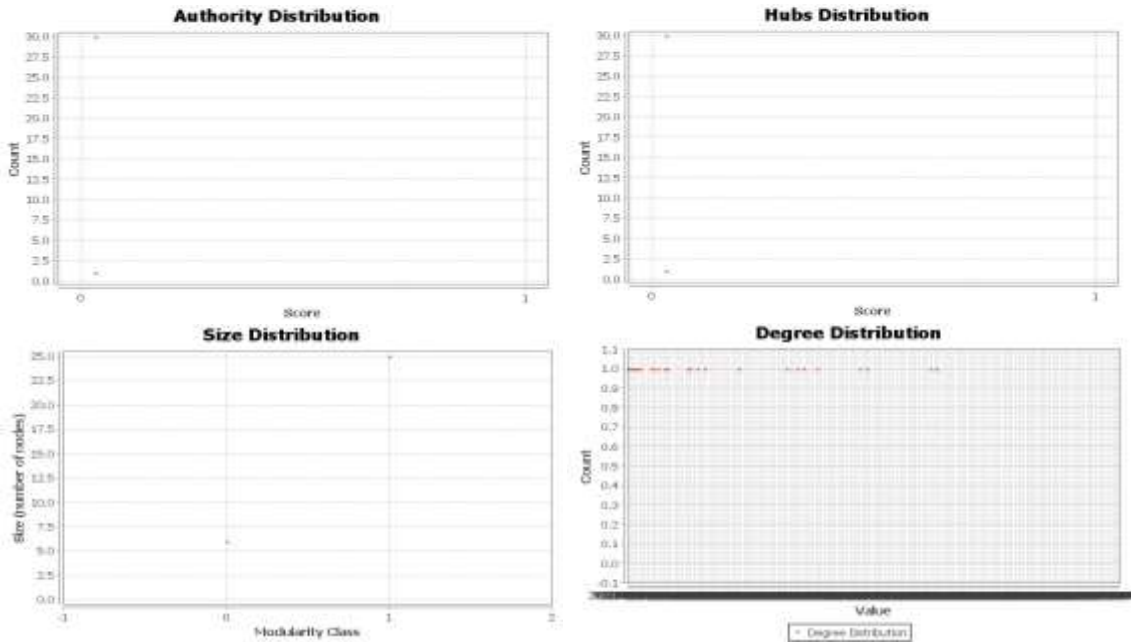
Figure 6: Centrality Distribution Metrics of EU Trade Network 2011

11. Eurozone Investment Network (EIN)

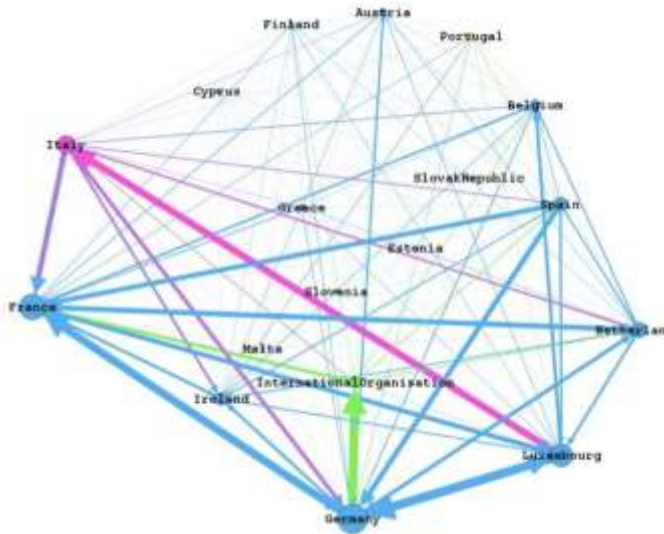*Figure 7: Eurozone Investment Network 2007(@CPIS2015)*

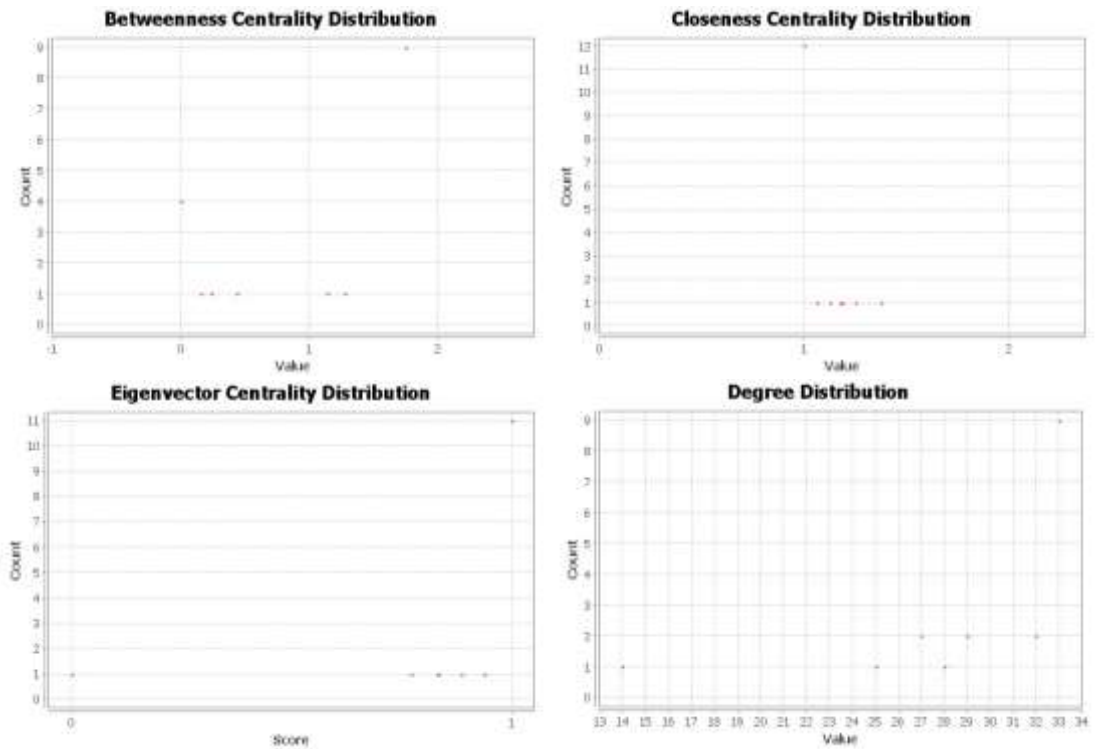*Figure 8: Centrality Distribution Metrics of Eurozone Investment Network 2007*
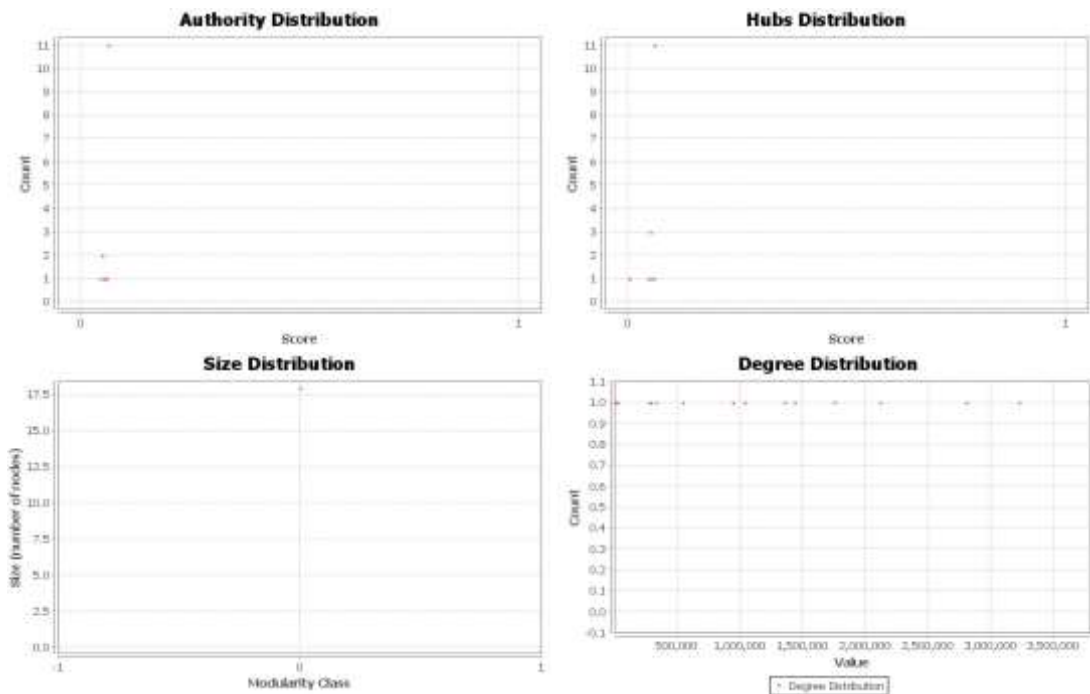

Figure 9: Centrality Distribution Metrics of Eurozone Investment Network 2007

Now we look at our investment network known as Eurozone Investment Network (EIN). The network consist 17 Eurozone members and 1 node of international organizations which won't represent a country in investment profiles. Figure 7 and 10 shows EIN of 2007 and 2011 respectively. Size of the node represent the weighted in and out degree, whereas different colors explains the similarities in connections between nodes. Large nodes are arranged as a cluster intentionally for better understanding of the complete graph. The countries and their respective connections are almost same in both networks. Though there is a visible change in weighted connections among strong nodes. To get better insights let's focus on centrality distributions of 2007 and 2011 EIN.

Some centralities measures remain same during both time periods, such as hubs and authority but unlike trade network we find lots of changes in EIN's centrality measures after financial crisis. It's loud and clear that whole investment scenario was changed afterwards. For example drastic reduction in betweenness centrality and increase in closeness centrality in 2011 as compared to 2007 shows that nodes are less resilient as well as having lower dependencies on other nodes. An overall reduction in degree centrality in 2011 sends a clear message of market contraction compared to 2007 levels, as edges are disappearing from the network. Whereas, increase in eigenvector centrality revels the creation of strong and stable connections with large

and central nodes, by other small or large nodes. This explains investors sentiments and expectations just after the crisis.

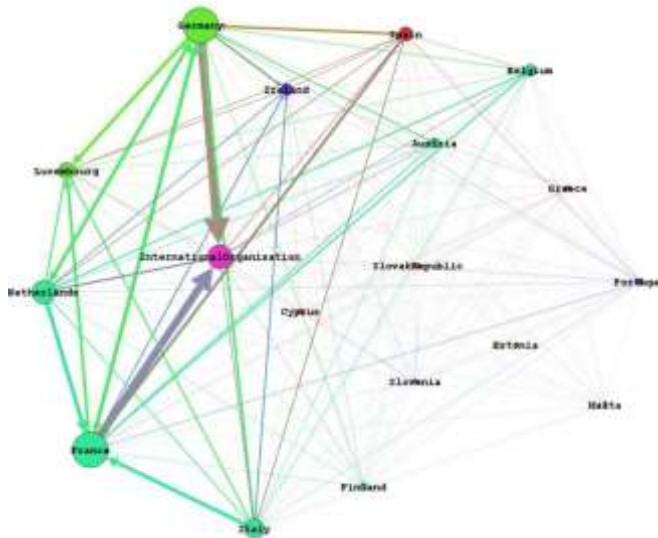*Figure 10: Eurozone Investment Network 2011(@CPIS2015)*

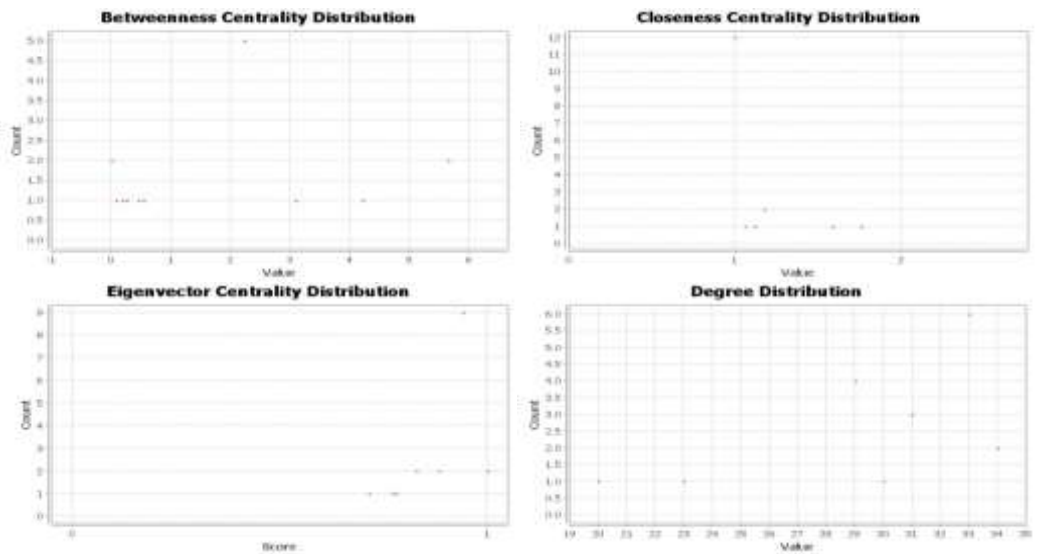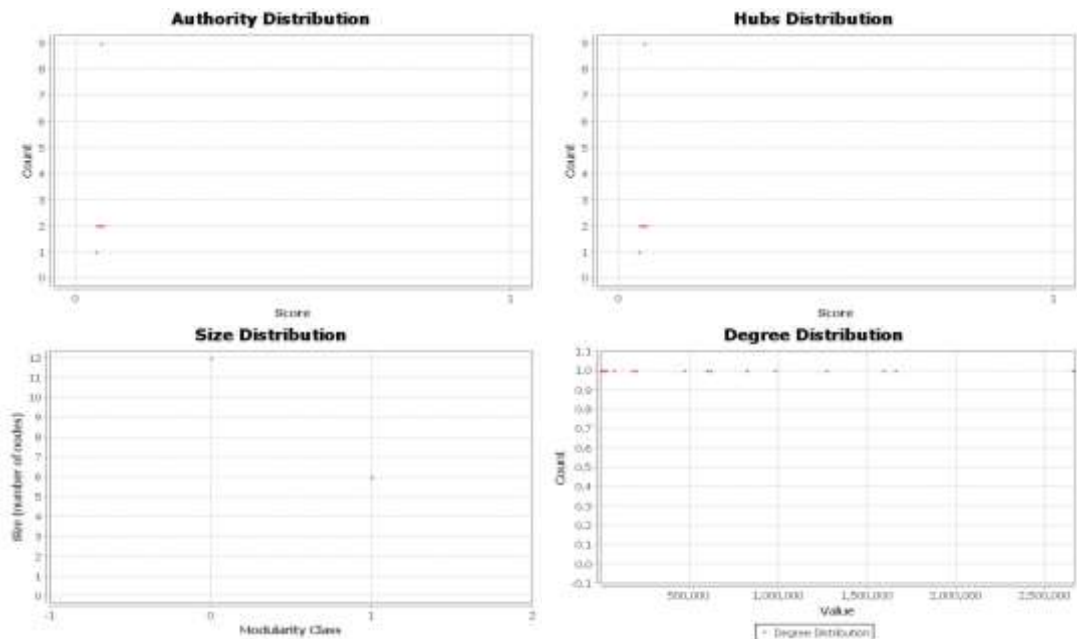*Figure 11: Centrality Distribution Metrics of Eurozone Investment Network 2011*

Figure 12: Centrality Distribution Metrics of Eurozone Investment Network 2011

Authority Distribution — **Hubs Distribution** — **Size Distribution** — **Degree Distribution**

12. Summary of Main Findings

1. Centrality measures are not effective for complete graphs and similar networks. As we have seen the results of trade network centrality measures. There is not much difference in all sorts of centrality measure used for the particular network during both time periods.

2. EU Trade Network (EUTN) shows strong structure and resilience. As the network and edges remains same before (2007) and after (2011) global financial crisis. Rather one can observe increase in weighted degree distribution.

3. Eurozone Investment Network (EIN) showed lots of changes in its structure and strength. After crisis network is not the same rather small, bit dispersed, and shaken. Investors lost their confidence on smaller markets as flow of funds is directed towards big economies, even small nodes are moving towards bigger and stronger nodes during 2011. Increase and decrease of different centrality measures explains the changes in 2007 and 2011 network in much better and understandable way.

4. As many centrality types were used to analyze different networks, we can observe that centrality measures can explain less dense and incomplete graphs or networks in much better way. Indeed one can have measures for all sorts of networks but dynamic graphs needs to have certain characteristics to be explained effectively by the used measures.

5. Another important observation is about the weights degree of both trade and investment networks. For trade networks average weighted degree is increased in 2011 compared to 2007.

While investment networks shows sign of weakness as there is a huge decline in weighted degree of after crisis (2011) network.*(See Table 1)*

6. Weighted degrees distribution of both networks shows the level of disproportion among nodes. This is uniform for both trade and investment networks, as few large nodes have huge in and out flows while rest of the networks shares small portion of liquidity and other flows.

	EUTN		EIN	
	2007	2011	2007	2011
Average Degree	29.968	29.968	15.000	15.222
Average Weighted Degree	177187.490	199814.377	554761.278	385768.111
Diameter	2	2	2	2
Radius	1	1	1	1
Average Path length	1.001	1.001	1.0657	1.1046
Number of shortest paths	930	930	289	306
Modularity	0.172	0.184	0.000	0.046
Modularity with resolution	0.172	0.184	0.000	0.046
Number of Communities	2	2	1	2
Number of Weakly Connected Components	1	1	1	1
Number of Strongly Connected Components	1	1	2	1
ECR Sum change	9.1944E-5	9.1944 E-5	1.7680 E-5	5.9869 E-5
HITS Parameter	E = 1.0E-4	E = 1.0E-4	E = 1.0E-4	E = 1.0E-4

Table1: Summery Statistics of EU Trade Network and Eurozone Investment Network

13. Conclusions

Centrality measures are used to analyse variety of networks. We used different local and relative centralities measures to analyse European Union's trade and investment networks. Analysed trade and investment networks correspond to time periods before and after global financial crisis which is 2007 and 2011. EU Trade Network is complete graphs having strong connections among its nodes. Trade networks for both years (2007 and 2011) remains identical with respect to centrality distributions. Similar results explain the limitations of centrality measures for complete graphs. Eurozone Investment Network is less dense rather not complete

structure. It showed huge changes in structure and resilience in its centrality distributions calculated for both years. Most of the measures used for analysis gave insights about investment scenarios and changes in investor's sentiments after global financial crisis

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Biographical notes: Muhammad Mohsin Hakeem is a MEXT fellow (Ministry of Education, Culture, Sports, Science and Technology, Japan) and doctoral student at Graduate school of economics and management, Tohoku University, Sendai, Japan.

Ken-ichi Suzuki is an Associate Professor at Graduate School of Economics and Management, Tohoku University, Sendai, Japan.